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7.2(a). $\varepsilon = cp$, $p = \frac{n\pi\hbar}{L}$, $\varepsilon = \frac{n\pi\hbar c}{L}$.

$D(n) = 4\pi n^2$

$$\frac{1}{8}(2)\int_0^{n_F} 4\pi n^2\, dn = N$$

$$\int_0^{n_F} \pi n^2\, dn = N$$

$$\frac{\pi}{3} n_F^3 = N, \quad n_F = \left(\frac{3N}{\pi}\right)^{1/3}.$$

$$\varepsilon_F = n_F \frac{\pi\hbar c}{L}$$

$$= \cancel{\frac{3N}{\pi}} \left(\frac{3N}{\pi V}\right)^{1/3} L \frac{\pi\hbar c}{L}$$

$$= \boxed{\pi\hbar c\left(\frac{3}{\pi}\frac{N}{V}\right)^{1/3}.}$$

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7.2 (b). Evaluating U_0 explicitly:

$$\frac{2}{8}\int_0^{n_F} 4\pi n^2\, dn\, \frac{\pi\hbar n c}{L}$$

$$= \int_0^{n_F} \frac{\pi^2 c\hbar}{L} n^3\, dn$$

$$= \frac{\pi^2 c\hbar}{4L} n_F^4.$$

recall
$$\boxed{\varepsilon_F = \frac{c\hbar\pi}{L} n_F}$$

$$= \frac{\pi^2 c\hbar}{4L}\left(\frac{L}{c\hbar\pi}\right)^4 \varepsilon_F^4$$

from part (a) $\Rightarrow$
$$= \frac{\pi}{4}\left(\frac{\pi c\hbar}{L}\right)\left(\frac{L}{c\hbar\pi}\right)^4 (\hbar c\pi)^3\,\frac{3}{\pi}\frac{N}{V}\,\varepsilon_F.$$

$$= \boxed{\frac{3}{4} N \varepsilon_F}$$

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